A numerical model of the shortbread baking process in a forced convection oven

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HIGHLIGHTS
- The evaporation of water had a significant effect on the temperature field.
- The numerical model associated the grade of browning with the temperature field.
- The results of the numerical and experimental grade of browning are comparable.
- The difference between the measured and simulated temperature at the oven was 2.8 K.

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ABSTRACT
The objective of all manufacturers and users of ovens is to achieve uniform browning of various baked foods. In recent years, manufacturers have found it difficult to achieve this, due to the rapid appearance of new trends and due to progressively shorter development times. In this paper, we present the development and validation of a time-dependent 3D computational fluid dynamics model, which enables the numerical prediction of the baking performance and grade of browning of a forced convection oven. Flow and heat transfer of hot air in an oven, where a round heating element and a fan are both operating, are simulated. Radiative and convective heat transfer is taken into account. We found, that it is necessary to include water evaporation in the model. The numerical model was validated by performing experimental measurements of temperature and by performing baking tests of shortbread. After baking, the grade of browning was measured for the shortbread. To determine the grade of browning, the method of identification of colour contrasts was used, based on the colour space CIE L*a*b. Based on the results, we proposed a linear model, which enabled the prediction of the grade of browning based on the results of the fluid dynamics simulation.

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1. Introduction
The market requirements for more and more advanced technologies and the follow-up of new design trends dictate a very dynamic development within the domain of cooking appliances. Manufacturers are faced with requirements for shorter time periods from the development of an idea to the launch of the product onto the market. This results in very short pre-development and development times. In most cases, the short development times also mean shorter times for the verification of the baking performance of the new oven design. Manufacturers and institutes verify the oven baking properties according to the standard EN 60350-1 [5].

In order to cope with short pre-development and development times, the use of numerical simulations, such as computational fluid dynamic (CFD) simulations, which help designers achieve uniform baking and temperature distribution, is becoming essential. The prediction of characteristics of ovens by means of numerical calculations during the pre-development phase, will present, in the years to come, one of the key tools in the design process. The aim of this paper is to develop and validate a numerical model, which can be used in the design process to predict the baking performance of household ovens.

In recent years, the understanding of the heat transfer phenomena in ovens for domestic use by means of numerical simulations have been studied by many authors [4,6–8,12,14,16]. The process of natural convection has been considered by researchers [4,6–8,17]. They established that radiation is the main mechanism of heat transfer when using the natural convection baking method.
The mechanism of forced convection has also been studied by several authors [12, 14–16]. In their work, when setting up a numerical simulation of the oven cavity, Rek et al. [12] did not take into consideration the fan of the oven and the heating element and so they were unable to prescribe precisely the boundary conditions at the openings in the fan cover. They established that the emissivity of the shortbread had no significant influence on the results of the numerical simulations, which is the result of the constant low temperature at the inlet into the oven cavity. Mistry et al. [7] developed a 3D time-dependent numerical model for simulations of natural convection within two different baking systems. By applying the time-dependent simulation, they succeeded in simulating the turning ON and OFF of the heating elements. In the case of the forced convection oven, they were able to predict the temperature field within 4% of the experimental value and in the case of the infrared operation of the upper heating elements, they found the difference between the simulation and the experiment to be 10%. The boundary conditions of the heating elements were set as a volume heat source. They established that the emissivity of the heating element had a greater effect on the temperature field in the oven cavity than the emissivity of the oven cavity walls. Verboven et al. [15, 16] applied the k-ε turbulent model in their research into the numerical model of forced convection in the oven cavity. The results of the average temperature in the oven differed by 4.6 °C between the simulation and the experiment, at a setting of 200 °C. By means of validation of the numerical model, they found that the key significance was presented by the performance capability of the fan, the shape of the fan, and the oven cavity geometry. On average, the error within the velocity field of the numerical model amounted to 22%.

The uniform browning of baked food is a very important indicator of how well the oven bakes. Researchers [3, 4, 8] used numerical simulations to study the bread baking process. In the majority of cases, they used a conventional baking system by adding vapour. The bread baking results were evaluated according to the grade of browning. Three different models of radiation were applied in the calculations, a discrete transfer radiation model, surface to surface, and discrete ordinates. All the stated radiation models gave similar results. The results were validated by applying the experimental measurements of the temperatures at different locations. Purlis and Salvadori [11] demonstrated that achieving the proper grade of browning is an important feature of an individual baking appliance. During their research, they applied the computerised determination of the load browning by means of the colour space CIE L’ab. They developed a mathematical model for the prediction of the grade of browning, based on the load weight loss and the oven baking temperature. They proved that the change in the browning of bread had a linear relationship with its weight loss. The image recording was carried out in natural light; which can cause significant deviations in the applications and validation of the results. Purlis [9] established that the development of browning is accompanied by the simultaneous process of mass transfer and heat transfer which, in most cases, act in non-ideal circumstances under non-ideal conditions. He also established that the process cannot be easily controlled and that the mechanism of chemical reactions taking place during the baking process has still not been studied enough. The browning of the load occurs due to various chemical reactions during the process of the treatment of the load. The key reactions are, above all, the caramelisation and the Maillard reaction. The sufficiently high enough temperature of the load, as defined by various authors, moves within the range between 105 °C and 120 °C, and the activity of water evaporation has to be met to start the baking procedure.

The objective of this research was to develop and validate models, which enable the numerical prediction of the baking performance of a forced convection oven. Firstly, we proposed a time-dependent 3D numerical model based on the numerical simulation of airflow and heat transfer. Secondly, we provided a simple model, which enables oven manufacturers to estimate the grade of browning based solely on the oven temperature. The numerical results were validated with experimental measurements of temperature, and by baking performance tests with shortbread. The shortbread was evaluated by the grade of browning with the measurement system based on the CIE L’ab colour space. The objective of this paper was to develop a numerical grade of browning, which is comparable with the experimental grade of browning and will replace expensive experimental measurements in the future. The paper is organised in the following way. In Section 2 we present the validation experiments. This is followed by Section 3, where the developed CFD model is presented. In Section 4, the results of simulations are presented and the models used to estimate the grade of browning based on CFD results are introduced. The last section summarizes the main results of the paper.

2. Experimental measurements

In order to be able to validate the developed numerical model and to estimate boundary conditions, we performed experimental measurements. The baking properties of the oven were tested according to the standard EN 60350-1 [5]. The experiment consisted of the measurement of temperature during baking and the grade of browning after the baking of shortbread. The baking of shortbread was chosen since measurement of the grade of browning of baked shortbread is a good indicator of oven performance. Design shortcomings, which result in uneven temperature or radiation distribution, are reflected very well on the grade of browning of baked shortbread.

During the baking process, we measured the temperature of the air in the centre of the oven as well as the temperature of the shortbread. The location of the temperature measurement in the shortbread was 2.5 mm from the baking tray in the middle of the shortbread. The experiments were performed at a stabilised voltage of 230 V by means of a stabiliser, manufactured by the company, Gorenje Orodjarna, d.o.o. The voltage, current, power, and consumption of energy were monitored by means of the Iskra MC740 multimeter. The collection of temperature data was carried out by using the measuring unit NI CompactDAQ 8-Slot Ethernet Chassis and NI cDAQ-9188 with the high-density thermocouple module NI 9213, manufactured by the company National Instruments. The collection of the temperature data was carried out by using a type J thermocouple (iron – constant), manufactured by Omega, with a

![Fig. 1. Experimental set-up of the temperature measuring system used to measure the temperature of the air in the oven and the temperature of the shortbread.](image-url)
measurement range from $-40$ °C to $+750$ °C. The experimental set-up is shown in Fig. 1.

During the baking process, the operation of the heating elements was recorded by power monitoring using an Iskra MC740 multimeter. Fig. 2 shows the heating element operation in an empty oven, the oven with a tray, and the oven with a tray and shortbread. The dimensions of the baking tray were 360 mm in length, 200 mm in width and 5 mm in height. The position of the baking tray was 10 mm from the fan cover and 40 mm below the centre of the oven. The differences in operation of the oven are due to the oven temperature control mechanism. The recorded values were used as time-dependent boundary conditions of the operation of the round heating element during numerical simulations.

The baking performance measurements were performed using shortbread and setting the oven system on hot air, which meant that the round heating element and the fan were both in operation. In addition to the procedure of the baking performance measurement, the standard EN 60350-1 [5] describes in detail the procedure of the preparation of the shortbread. The ingredients for the preparation of shortbread in mass portions are: 49.9% of white wheat flour, 19.9% of margarine, 19.9% of sugar, 10% of eggs, and 0.3% of salt. The total mass of all 9 shortbreads on the baking tray was 356 g. The dimensions of the shortbread on the baking tray were 100 mm in length, 20 mm in width and 5 mm in height. The thermal properties of the shortbread were calculated using the portions of the added ingredients, and are shown in Table 1, ASHRAE Handbook [2].

The baking process was performed until the required average grade of browning on the upper side of the shortbread $R_b = 43 \pm 5\%$ was reached, as defined by the standard. This duration was later reused in order to validate the simulation model. The measurements of the grade of browning were carried out at 63 points (matrix $7 \times 9$), by applying the Reference Browning Measurement System (RBMS) [13], Fig. 3. The measurement points were 35 mm away from the edge of the shortbread, while the distance between points was 55 mm. RBMS was developed for the needs of the EN 60350-1 standard for grading various kinds of meals prepared in ovens. This method is based on the optical measurement system, which records the shortbread surfaces of various shapes, to which the grade of browning is determined. The measurement system is composed of a chamber in which the lamps are installed, enabling constant conditions during the execution of the measurements. The measurement chamber was equipped with a CCD (Charge Coupled Device) camera, which recorded the shortbread surface. The Reference Browning Measurement System was manufactured in compliance with the ISO 7724 standard and CIE 15.2. The measurement system was based on the CIE L’a’b colour space. The measurements of the grade of browning were performed at a temperature of $20–25$ °C. The measurements of the shortbread browning $R_b$ were carried out one hour after baking.

The baking performance tests were carried out over 10 repetitions, where the achieved average grade of browning on the upper side was $R_b 43.26\%$. The average standard deviation of experimental measurements was 1.72. The baking lasted 780 s with the temperature in the middle of the oven set at 175 °C. The temperature and the test duration were used as boundary conditions during the numerical simulation. The results of one of the experimental tests are shown in Fig. 4.

3. CFD modelling

The numerical simulations were performed using the commercial software package ANSYS CFX 15 [1]. ANSYS CFX is a computer fluid dynamics (CFD) software, which has been used and validated for a wide variety of flow and heat transfer phenomena. It uses the finite volume method; the most widely spread approximation method for solving fluid dynamics and heat-transfer problems.

Radiation, conduction and convection heat transfer mechanisms were taken into account during the simulation of the baking process. The boundary conditions were defined by means of the already published research [6,7,12,14–16] and by means of the measurements of the experiment. The air in the oven cavity was treated as an ideal gas [3,4,7]. The air-flow inside the domain was assumed to be turbulent, with the Reynolds number being $9.7 \times 10^4$. The following Reynolds averaged Navier-Stokes equations for flow and heat-transfer in the oven cavity were solved:

the continuity equation
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)
\]

the momentum equation
\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}, \quad (2)
\]

and the energy equation
\[
\rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T, \quad (3)
\]

where $\mathbf{v}$ is the air velocity field, $p$ is the pressure and $T$ is the temperature. The density $\rho$ of the air was obtained by the ideal gas state equation, since due to high temperature differences in the oven, changes in air density are important. The thermal conductivity $k$ and the viscosity $\nu$ of air as well as the specific heat $c_p$ were considered to be constant. The solid material properties were also considered to be constant, as their variation with the temperature had a small effect on the baking process.

3.1. Domain set-up

The fluid simulation domain was the air in the oven cavity, while the solid domains were: the insulation of the oven cavity,
the door glass, the fan cover, the fan, the round heating element, the shortbread, and the baking tray with dimensions of 360 mm in depth and 450 mm in width. Fig. 5 shows all the domains, lists the materials and their emissivity. The dimensions of the 3D model were: width 500 mm, depth 450 mm, and height 380 mm.

The oven cavity fan with a diameter of 150 mm had 6 blades. At a temperature of 20 °C, the fan rotated at a speed of 1350 min⁻¹, and at a temperature of 175 °C it rotated at a speed of 1800 min⁻¹. This data was obtained by means of the experimental measurements using a digital stroboscope tachometer manufactured by Extech instruments. The fan was included within the rotating fluid domain of the numerical model which had a transient rotor stator interface model boundary condition within the stationary fluid domain. The numerical calculation was based on the assumption that the heating element had a volume energy source [7], at a power of 2100 W as was reported by EGO, the manufacturer and as were obtained by means of our measurements from the experiment. The two results of the experiment with an empty oven and the oven with a tray were used for validating the boundary conditions. The validated boundary conditions were then applied to more complex calculations with the shortbread. The numerical calculations were performed for 780 s over time steps of 10 s.

3.2. Simulation set-up

The presented numerical calculation used the SST turbulence model as it was more appropriate for coupled flow – heat transfer simulations within the complex geometry of the oven [1]. The radiation had an important role during the numerical calculation due to the high temperatures of the heating element which, during operation, reaches up to 700 °C. Optical thickness \(\alpha L = 0.0045\) is a good indicator for the selection of the radiation model. The length scale \(L = 0.45\) m and absorption coefficient 0.01 m⁻¹ gives the optical thickness \(<<1\). This means that models P1 and Rosseland are unsuitable for optically thin problems [1]. Thus, we chose the Monte Carlo radiation model as it enables simulation of transpar-
ent materials (the oven glass door). The interior elements, oven cavity, fan cover, and the baking tray are enamelled with the prescribed emissivity of 0.9 [6,7,12]. The heating element and the fan are made of steel with an emissivity of 0.85 [6,7].

3.3. Boundary conditions

The no-slip boundary condition was applied to the walls of the model [12,14]. The comparison between the results of the numerical simulations and the experimental measurements of the empty oven cavity helped us to identify the heat transfer coefficient at the insulation $h = 5 \text{ W/m}^2 \text{K}$ due to natural convection of outside air. We chose $h = 13 \text{ W/m}^2 \text{K}$ at the door glass due to the slightly forced convection of the cooling system. The conservative interface flux boundary condition was used for all connections between domains. In the numerical simulation, the glass was treated as a transparent medium with the absorption coefficient $a = 89.15 \text{ m}^{-1}$, calculated using Eqs. (4) and (5), and the refractive index of 1.51.

$$I = I_0 e^{-ax}.$$ \hspace{1cm} (4)

$$a = -\frac{1}{x} \ln(T) = 89.15 \text{ m}^{-1}.$$ \hspace{1cm} (5)

The enamelled domains (the oven cavity, the fan cover and the baking tray) are considered as composite materials of steel and enamel. Due to the limitation of the computer power and the complexity of the model, the numerical calculation did not consider the volume expansion of the shortbread, Chhanwal et al. [3], while the height of the shortbread, 5 mm was taken into account. The physical properties of the materials in use are listed in Table 1.

3.4. Treatment of water evaporation

As the evaporation of water takes place during baking, we measured the mass of the shortbread during the process. The results are presented in Fig. 6. During thermal treatment, the shortbread mass reduced by 14.89%. After 400 s of the baking process, when the whole shortbread reached 100 °C, the evaporation process may be considered to be approximately linear, with a gradient of 0.11 g/s.

Evaporation was considered in the numerical calculation as a heat sink on the shortbread. The heat sink was calculated with consideration of the water vapourisation heat $H_w$ of 2260 kJ/kg, the measured evaporation time $t = 380$ s, and the quantity of evaporated water $m = 42$ g. The evaporation was replaced by a sink of $mH_w/t = 249.7$ W. The influence of evaporation on the properties of air was neglected, as suggested by Chhanwal et al. [3].

3.5. Mesh analysis

The domains were meshed by tetrahedral mesh elements. The volumetric mesh had been refined towards the heating element, the fan cover, the glass door, and the fan. At the fan cover openings and the inner side of the glass, additional face sizing refinements were used.

The influence of computational mesh on the results of simulations was investigated by comparing the results of three different meshes having 2, 4, and 6 million elements. An empty oven was simulated. Time traces of temperature in the centre of the oven are compared with the results of the experiment in Fig. 8. The 2-million mesh demonstrated a more significant deviation in regard to the results of the experiment. As the difference between the mesh with 4 and 6 million elements was insignificant, we chose the medium 4-million mesh density for further simulations.
The mesh of the shortbread was structured using hexahedral elements made by the sweep method with 5 sweep elements. The fluid domain mesh had been refined at the shortbread.

3.6. Time-step analysis

The choice of time-step was confirmed by comparing simulations with three different time-steps, 1, 10, and 20 s. The time traces of temperature are presented in Fig. 9. The 20 s time-step demonstrated a more significant deviation with regard to the experimental results. This time-step is too long to adequately capture the ON and OFF phases of the heating element. The 10 s time-step was selected in order to decrease overall computational times. The average difference between temperatures of the experiment and simulated temperatures at the centre of the oven was 2.6 K when using the 10 s time-step.

4. Results and discussion

Fig. 10 shows the temperature time traces at the centre of the oven and in the shortbread. The average difference between the measured and simulated temperatures at the centre was 2.8 K and in the shortbread 5.6 K. In the first 380 s the heater was constantly turned on. The air temperature in the centre of the oven rose in an approximate linear manner. The temperature of the shortbread, however, exhibited a slower increase in temperature. This was due to the thermal properties of the shortbread, which prohibited fast heat conduction into the centre of the shortbread and at the same time due to the evaporation of water, which uses some of the available heat. After 380 s, the heater was turned on and off at approximately 60 s intervals. Due to the decreased input of heat into the oven, the temperature rose more slowly. The temperature measurement in the shortbread was carried out 2.5 mm from the tray on the middle shortbread piece.

In order to visualise the baking process, we show, in Fig. 11, the temperature contours of the shortbread at times of 530 s, 660 s, and 780 s after the start of the baking process. We observe a steady increase in temperature, and at the end of the baking process, higher temperatures in the corners of the oven.

In order to be able to estimate the grade of browning based on the CFD simulation results alone, we propose models, which connect the simulated temperature field, simulated received heat and the resulting grade of browning. First, we take into consideration that browning starts when the first measurement point on the shortbread exceeds 120 °C, which was established by Purlis and Salvadori [10]. Thus, based on simulation results, we chose the time of 530 s to start analysing the results.

We consider three cases. Firstly, we analysed the average temperatures at the shortbread by taking into account the results from 530 s to 780 s (the period when the first measurement point on the shortbread exceeds 120 °C). Secondly, we considered the instantaneous temperature field at the end of the baking process and thirdly, we considered the heat received by the shortbread in the period between 530 s and 780 s. The received heat was calculated by integration of wall heat flux over the surface of the shortbread and over time. Fig. 12 presents contours of the grade of browning, two temperature fields and the received heat. We observed an approximately linear relationship between the shortbread temperature $T$, heat $Q$ and the grade of browning $R_y$ and thus propose the following two models:

$$R_y = \alpha T + \beta,$$

$$R_y = \alpha' Q + \beta'.$$

Plotting the grade of browning versus the temperature and heat on a chart in Fig. 13, we were able to use the least squares method to find the constants of the models. The results, in the case of using average temperatures, were

$$\alpha = -0.4515 \ \text{C}^{-1} \ \text{and} \ \beta = 95.12 \ ^\circ\text{C}$$

in the case of instantaneous temperatures,
$a = -0.4041 \, ^\circC^{-1}$ and $\beta = 92.3 \, ^\circC$

and in the case of heat,

$a' = -0.0816 \, J^{-1}$ and $\beta' = 72.59 \, J$.

Based on a comparison of the results of the experiment and the numerical results, we developed linear models (6 and 7), which can be used to predict the grade of browning based on the CFD simulation of the oven according to the standard experiment described in EN60350-1 [5]. The proposed models can be used by measuring (or simulating) the temperature in the oven. Purlis and Salvadori [11] proposed a linear correlation between weight loss and oven baking temperature for the prediction of the grade of browning of baked bread. Since both our model and the model of Purlis and Salvadori [11] proposed a linear relationship, this can serve as additional validation of our results. Since our model does not require the measurement of weight loss, it is easier to apply during the design process of a new oven.

We predicted the grades of browning using models (6 and 7) and plotted them alongside the measured values in Fig. 14. Furthermore, we estimated the difference between the measured and the predicted grades of browning using the normalized root mean square deviation:

$$rms = \frac{\sqrt{\sum (R_{\text{exp}} - R_{\text{sim}})^2}}{\sqrt{\sum R_{\text{exp}}^2}}.$$  

We observed that using the instantaneous temperature field at the end of the baking process or using the average temperature field produces results of similar quality ($rms = 0.02629$ for averaged temperatures and $rms = 0.02639$ for instantaneous temperature). The model that uses received heat, yields a poorer agreement ($rms = 0.03276$). The grade of browning differ from the measured values for less than the standard deviation of the measurements of the experiment. The reason that the model, which uses received heat, does not match the result of the experiment well, is because the heat is calculated using an integral over the whole shortbread and is not locally focused around a single point, as is the case with temperature measurement and grade of

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**Fig. 11.** Shortbread temperatures at (a) 530 s, (b) 660 s, and (c) 780 s.

**Fig. 12.** Contours of (a) measurements of the experiment on the grade of browning, (b) simulated averaged temperature field, (c) simulated instantaneous temperature field at the end of the baking process and (d) simulated heat input.

**Fig. 13.** Relationship between the grade of browning and (a) average temperature, (b) instantaneous temperature, (c) heat input. Linear models (6 and 7) are also shown.
browning measurement. Based on these findings, we conclude that the models, which use the CFD simulated temperature of the shortbread, are capable of predicting the grade of browning.

5. Conclusions

A numerical model was developed, which enables the prediction of the grade of browning of oven baked goods based on the results of a CFD simulation. The numerical model was validated by means of measurements of temperature in the oven during the baking of shortbread and by measuring the grade of browning of baked shortbread. The measurements of the experiment of baking shortbread were performed according the standard EN 60350-1 [5]. The results of numerical calculations were validated with different mesh densities and different time step settings. The proposed model, which uses the simulated average temperature field of the shortbread to predict the grade of browning, matches the experimental observations very well. It was established that the temperature field has a direct effect on the grade of browning of the shortbread. A model was developed, based on the least squares method, where we observed a linear relationship between the numerical calculated shortbread temperature \( T \) and experimental measured grade of browning \( R_y \). For the use of the model in forced convection ovens, the \( \alpha \) and \( \beta \) constants were defined.

The purpose of the developed numerical model was to help designers to avoid making costly experimental measurements of prototype ovens. The developed numerical model for the prediction of the baking properties of oven cavities will enable, in the pre-development and development phases of design, the prediction and amelioration of the oven cavity design. The model may be used for the definition or improvement of the shape of the oven cavity, oven fan, baking levels, shapes of the heating elements, and the openings in the fan cover of the oven cavity. Furthermore, the model may be used to research and improve the energy consumption of oven cavities. In this paper, we have established that the use of a water evaporation model in numerical calculation, has a significant effect on the temperature field and the time course of the temperature in the shortbread and in the oven cavity, and cannot be neglected. The objective of the application of the numerical simulations in the engineering practice is to provide quick solutions and enable important flexibility. The advantage of numeric models in the engineering practice is to provide quick solutions that cannot be neglected. The simplified numerical simulations enable the solving of complex engineering issues over a very short time.

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[1] ANSYS 15 CFX, ANSYS Inc., Southpointe, 2600 ANSYS Drive, Canonsburg, PA 15317, USA.

Fig. 14. Contours of (a) the experimental grade of browning, (b) numerically predicted grade of browning based on the average temperature field, (c) numerically predicted grade of browning using the temperature field at the end of baking process, and (d) numerically predicted grade of browning using heat.